How do Brent crude oil trends affect the Moroccan stock market?

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Abstract. This study explores the impact of Brent crude oil market trends on the Moroccan stock market, focusing on both the overall MASI index and its sector-specific MASI Oil index, using Multifractal Detrended Asymmetric Cross-Correlation Analysis (MF-ADCCA). The initial analysis of cross-correlations between Brent and the MASI/MASI OIL indices revealed significant long-range dependencies, indicating a strong interconnection between Brent and the two indices. Asymmetric DCCA cross-correlation coefficients further revealed asymmetric and persistent cross-correlations across different time scales, emphasizing the long-range dependencies between Brent and the two indices under varying market conditions. The MF-ADCCA analysis, which included generalized Hurst exponents, Rényi exponents, and singularity spectrum functions, showed that the cross-correlations exhibited multifractal behavior across all orders of fluctuations, highlighting the complexity of their interactions. In the monofractal case, the analysis of uptrend and downtrend fluctuation functions demonstrated distinct behaviors across all time scales, reinforcing the presence of asymmetry in the cross-correlations between Brent and the two Moroccan indices. Additionally, the generalized Hurst exponents for uptrend and downtrend periods showed nonlinear decreases throughout the fluctuation range, further signifying multifractal characteristics during both bullish and bearish market phases. The graphical analysis confirmed that the uptrend and downtrend generalized exponents exhibited distinct patterns, surrounding the overall generalized Hurst exponent, indicating that correlations between Brent and the indices vary depending on the direction of the Brent market trend. These findings highlight the asymmetric nature of the cross-correlations, which are contingent on market conditions. The implications for investors, fund managers, policymakers, and Morocco's economic resilience are significant, offering insights into navigating oil price volatility and strengthening financial stability.
Keywords: *Brent crude oil future, MASI index, MASI oil index, Cross-correlation, Multifractality, Asymmetry, MF-ADCCA.*

1. Introduction

The interplay between oil prices and stock markets is a critical topic of investigation for economists and financial analysts, owing to oil's central role in the global economy. Oil price volatility affects a wide range of macroeconomic indicators, such as inflation, industrial output, and exchange rates, which in turn can influence stock market performance. While existing research has thoroughly explored the impact of oil prices on stock markets—particularly in oil-importing and oil-dependent economies, the complexity of these relationships necessitates more advanced analytical methods to capture their non-linear and dynamic nature.

Morocco, as a growing economy in North Africa, relies heavily on oil imports to meet its energy demands, making it particularly vulnerable to fluctuations in global oil prices. Brent Crude Oil, one of the primary benchmarks for global oil pricing, is a significant factor in determining energy costs in Morocco, particularly for refined petroleum products such as gasoline and diesel. Changes in Brent prices can ripple through the Moroccan economy, affecting inflation, the trade balance, and broader economic growth.

This sensitivity is especially evident in key sectors such as transportation, manufacturing, and agriculture, where fuel costs directly influence operational expenses. Given Morocco's dependence on imported petroleum products, movements in Brent prices often result in domestic fuel price adjustments, which have an impact on both consumers and businesses. Morocco's stock market, represented by indices such as the MASI and the MASI Oil & Gas Index, tends to reflect the impact of these fluctuations, particularly in oil-related industries and sectors.

Understanding the influence of oil price movements on Morocco's stock market, particularly in the context of its reliance on imported energy, is essential for both investors and policymakers. Traditional models that analyze the relationship between oil prices and stock markets often assume a linear correlation, overlooking the possibility of more complex, non-linear dependencies. Such methods may not fully capture the long-term, multifractal, or asymmetric cross-correlations that exist between these markets.

To address these gaps, this study applies the Multifractal Detrended Asymmetric Cross-Correlation Analysis (MF-ADCCA), a robust tool designed to uncover both multifractal properties and asymmetries in cross-correlations between financial time series. The MF-ADCCA model provides a deeper understanding of how markets behave differently under bullish versus bearish trends, revealing the persistence of long-term cross-correlations and asymmetries between oil prices and stock markets. This makes it an ideal approach for examining the intricacies of the oil-stock market relationship, particularly in Morocco, where energy dependence is a key factor.

The primary goal of this research is to analyze the multifractal and asymmetric cross-correlations between Brent Crude Oil prices and the Moroccan stock market, focusing on both the overall MASI index and its sector-specific MASI Oil index. By leveraging the MF-ADCCA approach, this study seeks to shed light on the varying degrees of dependence, asymmetry, and persistence between Brent Crude Oil and Moroccan stock market.

The originality and contribution of this paper stem from both its contextual focus and methodological approach. From a contextual perspective, unlike most existing research that centers on large oil-exporting or oil-importing economies, this study focuses on the Moroccan market—a relatively underexplored yet crucial case due to its significant reliance on imported petroleum products. By analyzing both the MASI index and the MASI Oil & Gas sectoral index, the paper offers a more detailed insight into how oil price fluctuations affect different segments of Morocco's stock market. Methodologically, the study applies the Multifractal Detrended Asymmetric Cross-Correlation Analysis (MF-ADCCA), an advanced technique that effectively captures long-range dependencies, multifractality, and asymmetries in cross-correlations—characteristics often overlooked by traditional linear models. To the best of our knowledge, this work represents the first application of MF-ADCCA to the Moroccan market within the context of oil-stock relationships, thereby addressing a significant gap in the literature. The results will provide valuable insights for investors seeking to manage risks and optimize portfolio diversification, as well as for policymakers aiming to design informed economic policies.

This paper is structured as follows: Section 2 presents a review of the relevant literature. Section 3 describes the methodology and data used in the study. Section 4 compares the results of this study with the previous ones, while Section 5 provides conclusions and outlines the implications of the findings.

2. Literature review

The relationship between oil price shocks and stock market returns has been widely explored using a range of traditional econometric approaches, such as VAR, VECM, DCC-GARCH-GJR, and structural VAR models. These methodologies have offered meaningful insights into the complex interplay between oil markets and equity performance. For instance, Filis et al. (2011) utilized DCC-GARCH-GJR models to investigate the time-varying nature of the oil-stock market relationship, revealing that precautionary demand shocks were associated with negative correlations, whereas aggregate demand shocks tended to generate positive correlations. In another study, Cuñado and Pérez de Gracia (2014) employed VAR and VECM frameworks to assess the impact of oil price shocks on stock returns in oil-importing European countries, concluding that supply-side shocks exerted a more pronounced negative effect than demand-side shocks. Similarly, Fang and You (2014) used a structural VAR model to examine oil price shocks in China, India, and Russia, demonstrating that the effects of such shocks differed depending on each nation's economic structure and level of oil dependency.

Other econometric models have been applied to explore the interconnectedness between oil prices and stock market indices. For instance, Gomez-Gonzalez et al. (2020) utilized LASSO methods to investigate the link between Brent crude oil prices and stock indices in oil-dependent economies during financial crises. Hwang and Kim (2021) found that demand-driven oil price shocks had stronger and more persistent effects on stock markets during recessions.

In addition to econometric approaches, wavelet and causality-based techniques have been employed to further understand the oil-stock market nexus. Jammazi et al. (2017) used wavelet analysis and dynamic causality tests to examine time-varying causal relationships between oil prices and stock returns, noting that bidirectional causality intensified during the Global Financial Crisis. Khalfaoui et al. (2019) explored volatility spillovers between oil and stock markets in oil-importing and oil-exporting countries, emphasizing the greater vulnerability of oil-importing nations to lagged oil price shocks.

Beyond traditional approaches, sector-specific studies have provided additional insights into the uneven impact of oil price shocks. Nandha and Faff (2008) examined 35 global industry indices and found that rising oil prices negatively affected most industries, except for those directly linked to oil. Rahman (2022) used a nonlinear bivariate SVAR model to show that oil price volatility plays a critical role in shaping U.S. firms' investment decisions.

Recent advancements have been made in predicting volatility and cross-correlation behavior between oil and stock markets. Yang et al. (2019) employed the DCC-MIDAS method to examine long-term correlations between crude oil and stock markets, discovering that macroeconomic factors such as the risk-free rate and economic activity were key drivers of long-term correlations. Ivanovski and Hailemariam (2021) applied the Generalized Autoregressive Score (GAS) model to forecast volatility and correlations, finding significant variations during periods of economic stress. Ben-Salha and Mokni (2022) combined Detrended Cross-Correlation Analysis (DCCA) with a quantile-based approach, showing that cross-correlations between WTI crude oil and the S&P 500 index were highly sensitive to market conditions.

A growing body of research has applied multifractal methods to analyze the nonlinear and multifractal nature of cross-correlations in financial markets. Burugupalli (2015) utilized Multifractal Detrended Cross-Correlation Analysis (MF-DCCA) to investigate cross-correlations between gold and WTI crude oil, revealing strong multifractal correlations in the short term that weakened over time. Jianfeng et al. (2016) applied MF-DCCA to investigate cross-correlations between crude oil and exchange rates, uncovering multifractal patterns often missed by traditional econometric models. Yao et al. (2020) used MF-DCCA to assess the nonlinear relationships among economic policy uncertainty, crude oil, and stock markets, identifying persistent multifractal correlations. Adekoya et al. (2023) demonstrated how oil price shocks during the Russia-Ukraine war amplified multifractal behavior in European stock markets.

The use of multifractal methods has also been extended to the exploration of asymmetric cross-correlations. Gajardo and Kristjanpoller (2017) applied Multifractal Asymmetric Detrended Cross-Correlation Analysis (MF-ADCCA) to analyze cross-correlations between Latin American and U.S. stock market indices and the crude oil market. Their findings indicated the presence of multifractality in these cross-correlations, with varying degrees of asymmetry based on the market trends considered. Similarly, Mensi et al. (2021) examined the asymmetric multifractality of stock markets in major crude oil-producing and consuming countries, revealing stronger multifractality in upward market movements, except for China.

More recent studies have focused on the effects of major events such as the COVID-19 pandemic on oil-stock market dynamics. Mobeen et al. (2022) explored the bidirectional spillover effects between socially responsible investments (SRI) and oil markets before and during the pandemic using an asymmetric multifractal detrended approach. Their findings showed that SRI funds exhibited strong hedging capabilities, particularly during the pandemic, and provided notable diversification benefits under bearish market conditions.

Additional studies have explored the asymmetric relationships between energy markets and financial markets. Si-Min and Hong-Yong (2023) analyzed the cross-correlations between the Chinese energy futures market and energy stock markets, finding heightened multifractality after the COVID-19 outbreak. Meijun and Cao (2023) examined the asymmetric cross-correlations between crude oil markets (INE, WTI, Brent) and Chinese financial markets, highlighting significant volatility spillovers from oil markets to financial markets, especially during periods of market stress. Alharbey et al. (2023) explored the asymmetric volatility spillovers between the oil market and renewable energy stock markets in the U.S., Europe, and Asia, offering valuable insights for investors regarding portfolio design and risk management.

This study complements and extends the existing literature by adopting the asymmetric multifractal detrended cross-correlation analysis (MF-ADCCA) to examine the complex and time-varying relationship between oil price shocks and stock market returns. While traditional econometric and multifractal methods have provided significant insights into the linear, nonlinear, and scale-dependent dynamics of this relationship, the asymmetric multifractal approach adds a new dimension by capturing directional asymmetries in cross-correlations under varying market conditions. This methodological innovation allows for a more nuanced understanding of how oil price shocks differently affect stock markets during bullish and bearish phases, thereby offering deeper implications for risk management, policy formulation, and investment strategy development in increasingly volatile and interconnected global markets.

3. Methodology and data

3.1 Methodology

In this section, we describe three methods: The MF-ADCCA method, the Q-Cross-correlation significance test and the DCCA coefficient method.

3.1.1 Description of the MF-ADCCA

Cao et al. (2014) introduced the Multifractal Detrended Asymmetric Cross-Correlation Analysis (MF-ADCCA), a method that integrates the principles of Multifractal Detrended Cross-Correlation Analysis (MF-DCCA) with those of Asymmetric Multifractal Detrended Fluctuation Analysis (A-MFDFA). This approach captures multifractality—characterized by the presence of multiple scaling exponents within a time series—revealing the intricate and heterogeneous nature of its fluctuations.

Detrended Fluctuation Analysis (DFA), originally introduced by Peng et al. (1994), was designed to detect long-range correlations in non-stationary time series. Building on this foundation, Kantelhardt et al. (2002) developed Multifractal Detrended Fluctuation Analysis (MF-DFA), which allows for the exploration of multifractal properties across different time scales. To examine interactions between two time series, Podobnik and Stanley (2008) proposed Detrended Cross-Correlation Analysis (DCCA). This approach was further refined by Zhou (2008), who introduced Multifractal Detrended Cross-Correlation Analysis (MF-DCCA), combining the techniques of DCCA and MF-DFA. Subsequently, in 2009, Alvarez-Ramirez et al. extended the DFA framework by introducing Asymmetric Multifractal Detrended

Fluctuation Analysis (A-MFDFA), a method specifically designed to capture asymmetries in the scaling behavior of time series data.

Let $x = (x(k))_{1\leq k\leq N}$ and $y = (y(k))_{1\leq k\leq N}$ be two time series of length N. In this study, the series x represents the logarithmic returns of Brent Oil Futures (BRENT), while y corresponds to the logarithmic returns of one of two Moroccan stock indices: the overall MASI index or the MASI Oil & Gas sectorial index (hereafter referred to as MASI Oil). It is assumed that both series have compact support, meaning that $x(k) = 0$ and $y(k) = 0$ for only an insignificant fraction of values k, thus ensuring the data are effectively non-zero across most of the time span.

Step 1: For the time series x and y, we construct their respective profiles $X = (X(i))_{1\leq i\leq N}$ and $Y = (Y(i))_{1\leq i\leq N}$ as follows:

$$X(i) = \sum_{k=1}^{N}(x(k) - \bar{x}) \qquad\qquad Y(i) = \sum_{k=1}^{N}(y(k) - \bar{y}) \tag{1}$$

where $\bar{x}$ and $\bar{y}$ denote the mean values of the time series x and y, respectively.

Step 2 : For each time scale s, we divide the two profiles X and Y and the first series x, into $N_s = Int(N/s)$ non-overlapping sub-time series of the same length s, where $Int(.)$ gives the integer part of a real number. Based on the recommendations of Peng et al. (1994), $5 \leq s \leq N/4$ is traditionally selected. Since N is generally not a multiple of s, a short part at the end of the profiles may be neglected. To incorporate all the ignored parts of the series, the same procedure is repeated starting from the end of the profile. Thus, we obtain $2N_s$ intervals $I_{v,s} = \left(I_{v,s}(j)\right)_{1\leq j\leq s}$ defined by:

For each time scale s, the profiles X and Y, as well as the original time series x, are divided into $N_s = Int(N/s)$ non-overlapping segments of equal length s, where $Int(.)$ denotes the integer part of a real number. Following the guidelines of Peng et al. (1994), the scale s is typically chosen such that $5 \leq s \leq N/4$. Since N is not necessarily a multiple of s, a short segment at the end of the profiles may be left out. To account for these discarded portions, the same segmentation process is repeated starting from the end of each profile. As a result, a total of $2N_s$ segments $I_{v,s} = \left(I_{v,s}(j)\right)_{1\leq j\leq s}$ are obtained, defined as follows:

$$\begin{aligned} I_{v,s}(j) &= (v-1)s + j \qquad \text{for } v = 1,2,\cdots,N_s \\ I_{v,s}(j) &= (N - v - N_s)s + j \qquad \text{for } v = N_s + 1,2,\cdots,2N_s \end{aligned} \tag{2}$$

We denote by $X_{v,s}$, $Y_{v,s}$ and $x_{v,s}$, the v^{th} sub-time series corresponding to X, Y and x, respectively:

$$\begin{gathered} X_{v,s}(j) = X\big((v-1)s + j\big) \qquad Y_{v,s}(j) = Y\big((v-1)s + j\big) \\ x_{v,s}(j) = x\big((v-1)s + j\big) \end{gathered} \tag{3}$$

for $v = 1,2,\cdots,N_s$ and:

$$\begin{gathered} X_{v,s}(j) = X\big((N - v - N_s)s + j\big) \qquad Y_{v,s}(j) = Y\big((N - v - N_s)s + j\big) \\ x_{v,s}(j) = x\big((N - v - N_s)s + j\big) \end{gathered} \tag{4}$$

for $v = N_s + 1,2,\cdots,2N_s$.

Step 3: For each time scale s and for each segment $v = 1,2,\cdots,2N_s$, we estimate the local trends $\tilde{X}_{v,s}$ and $\tilde{Y}_{v,s}$ by performing a degree-2 polynomial least-square regressions of the sub-time series $X_{v,s}$ and $Y_{v,s}$ on the interval $I_{v,s}$:

$$\tilde{X}_{v,s}(j) = \alpha_0^{v,s} + \alpha_1^{v,s}.j + \alpha_2^{v,s}.j^2 \qquad \tilde{Y}_{v,s}(j) = \beta_0^{v,s} + \beta_1^{v,s}.j + \beta_2^{v,s}.j^2 \tag{5}$$

Simultaneously, we measure the local linear least-squares fits $\tilde{x}_{v,s}$ by performing linear least-square regressions of the sub-time series $x_{v,s}$ over the interval $I_{v,s}$:

$$\tilde{x}_{v,s}(j) = a^{v,s} + b^{v,s}.j \tag{6}$$

We then calculate the detrended covariances:

- For $v = 1,2,\cdots,N_s$:

$$f_{XY}^2(v,s) = \frac{1}{s}\sum_{j=1}^{s}\left|X\big((v-1)s+j\big) - \tilde{X}_{v,s}(j)\right|.\left|Y\big((v-1)s+j\big) - \tilde{Y}_{v,s}(j)\right|$$

- For $v = N_s + 1,2,\cdots,2N_s$: (7)

$$f_{XY}^2(v,s) = \frac{1}{s}\sum_{i=1}^{s}\left|X\big((N-v-N_s)s+j\big) - \tilde{X}_{v,s}(j)\right|.\left|Y\big((N-v-N_s)s+j\big) - \tilde{Y}_{v,s}(j)\right|$$

Step 4:

➢ The q^{th} order overall fluctuation functions:

For each time scale s and for a given order q, the q^{th} order overall fluctuation function $F_q(s)$ is defined by the MF-DCCA method (Zhou, 2008) as an average of the covariances over all segments by:

For $q \neq 0$: $$F_q(s) = \left[\frac{1}{2N_S}\sum_{v=1}^{2N_S}\big(f_{XY}^2(v,s)\big)^{\frac{q}{2}}\right]^{\frac{1}{q}}$$

For $q = 0$: $$F_0(s) = exp\left[\frac{1}{4N_S}\sum_{v=1}^{2N_S} ln\big(f_{XY}^2(v,s)\big)\right] \tag{8}$$

➢ The q^{th} order uptrend and downtrend fluctuation functions:

We consider the asymmetric cross-correlation between two markets represented by x and y under different market trends of the first time series x. For each time scale s and for a given order q, the q^{th} order uptrend fluctuation function $F_q^+(s)$ and the q^{th} order downtrend fluctuation function $F_q^-(s)$ are obtained by averaging the covariances $f_{XY}^2(v,s)$ over all segments v, when the sign of the slope $b^{v,s}$ is strictly (positive) (resp. negative):

For $q \neq 0$:

$$F_q^+(s) = \left[\frac{1}{M^+}\sum_{v=1}^{2N_S}\frac{1+sign(b^{v,s})}{2}\big(f_{XY}^2(v,s)\big)^{\frac{q}{2}}\right]^{\frac{1}{q}}$$

$$F_q^-(s) = \left[\frac{1}{M^-}\sum_{v=1}^{2N_S}\frac{1-sign(b^{v,s})}{2}\big(f_{XY}^2(v,s)\big)^{\frac{q}{2}}\right]^{\frac{1}{q}} \tag{9}$$

and:

For $q = 0$:

$$F_0^+(s) = exp\left[\frac{1}{2M^+}\sum_{v=1}^{2N_S}\frac{1+sign(b^{v,s})}{2} ln\big(f_{XY}^2(v,s)\big)\right]$$

$$F_0^-(s) = exp\left[\frac{1}{2M^-}\sum_{v=1}^{2N_S}\frac{1-sign(b^{v,s})}{2} ln\big(f_{XY}^2(v,s)\big)\right] \tag{10}$$

where $M^+ = \sum_{v=1}^{2N_S}\frac{1+sign(b^{v,s})}{2}$ and $M^- = \sum_{v=1}^{2N_S}\frac{1-sign(b^{v,s})}{2}$ represent the number of sub-time series $x_{v,s}$ with positive and negative trends, respectively. We assume $b^{v,s} \neq 0$ for all $v =$

$1,2,\cdots,2N_s$, such that $M^+ + M^- = 2N_s$.

Step 5: The scaling behavior of the fluctuations

If x and y are long-range power-law cross-correlated, then the q^{th} order fluctuation functions will behave, for sufficiently large values of s, according to the following power-law scaling law:

$$F_q(s) \sim s^{H_{XY}(q)} \qquad F_q^+(s) \sim s^{H_{XY}^+(q)} \qquad F_q^-(s) \sim s^{H_{XY}^-(q)} \tag{11}$$

where $H_{XY}(q)$, $H_{XY}^+(q)$ and $H_{XY}^-(q)$ are called the overall generalized Hurst exponent, the uptrend generalized Hurst exponent and the downtrend generalized Hurst exponent, respectively.

To estimate the values of $H_{XY}(q)$, $H_{XY}^+(q)$ and $H_{XY}^-(q)$ for different values of q, we perform a log-log linear Ordinary Least Squares regression of the time series $H_{XY}(q)$, $H_{XY}^+(q)$ and $H_{XY}^-(q)$ on the time series $F_q(s)$, $F_q^+(s)$ and $F_q^-(s)$, respectively.

The interpretation of $H_{XY}(q)$ follows the procedure outlined in the MF-DCCA method. When $H_{XY}(q)$ depends on q, the cross-correlation of x and y is multifractal, otherwise it is monofractal. In the case of $q = 2$, the exponent $H_{XY}(2)$ is interpreted in the same manner as the Hurst exponent. If $H_{XY}(2) > 0.5$, the cross-correlation of x and y are said to be persistent. When $H_{XY}(2) < 0.5$, the cross-correlation is said to be anti-pesistent. The case $H_{XY}(2) = 0.5$ indicates a short-range or no cross-correlation.

$H_{XY}(q)$ is a decreasing function and to measure the intensity of multifractality between x and y, we can use ΔH_{XY} defined by:

$$\Delta H_{XY} = H_{XY-Max} - H_{XY-Min} = H_{XY}(q_{min}) - H_{XY}(q_{max}) \tag{12}$$

The larger ΔH_{XY} is, the stronger the degree of multifractality will be.

It is well known that $H_{XY}(q)$ is directly related to the multifractal scaling exponent $\tau_{XY}(q)$, commonly known as the Rényi exponent:

$$\tau_{XY}(q) = q.H_{XY}(q) - 1 \tag{13}$$

If $\tau_{XY}(q)$ increase nonlinearly with q, the cross-correlation of the two series is multifractal. Otherwise, if $\tau_{XY}(q)$ is a linear function of q, then the cross-correlation is monofractal.

Another interesting way to characterize the multifractality of the cross-correlations, is to use the singularity spectrum $f_{XY}(\alpha_{XY})$ of the Hölder exponent α_{XY}. It is well known that the singularity spectrum $f_{XY}(\alpha_{XY})$ is related to the Rényi exponent $\tau_{XY}(q)$ through the Legendre transform:

$$\begin{cases} \alpha_{XY} = \tau'_{XY}(q) \\ f_{XY}(\alpha_{XY}) = q.\alpha_{XY} - \tau_{XY}(q) \end{cases} \tag{14}$$

where $\tau'_{XY}(q)$ is the derivative of the function $\tau_{XY}(q)$.

When the cross-correlation between the two series is multifractal, then the singularity spectrum $f_{XY}(\alpha_{XY})$ present a concave bell-shaped curve.

The richness of the multifractality can be determined by the width of the spectrum as:

$$\Delta\alpha_{XY} = \alpha_{XY-max} - \alpha_{XY-min} \tag{15}$$

The wider the spectrum, the richer the multifractal behavior of the cross-correlation of the analyzed time series.

We can deduce the relationship between $h(q)$ and $f_{XY}(\alpha_{XY})$ by:

$$\begin{cases} \alpha_{XY} = H_{XY}(q) + q.H'_{XY}(q) \\ f_{XY}(\alpha_{XY}) = q.\left(\alpha_{XY} - H_{XY}(q)\right) + 1 \end{cases} \tag{16}$$

By analogy, we can define the uptrend Rényi exponent $\tau_{XY}^+(q)$, the downtrend Rényi exponent $\tau_{XY}^-(q)$, the uptrend singularity spectrum $f_{XY}^+(\alpha)$ and the downtrend singularity spectrum $f_{XY}^-(\alpha)$, when the first market has different trend.

The analysis of the asymmetry of the cross-correlation results from the comparison of $H_{XY}^+(q)$ and $H_{XY}^-(q)$. If $H_{XY}^+(q) = H_{XY}^-(q)$, the cross-correlation between x and y is symmetric.

Conversely, if $H_{XY}^{+}(q) \neq H_{XY}^{-}(q)$, the cross-correlation between the x and y is asymmetric, meaning that the cross-correlation is different when the trending of x is positive than when it is negative. Thus, to measure the asymmetric degree of the cross-correlations, we use:

$$\Delta H_{XY}^{\pm}(q) = H_{XY}^{+}(q) - H_{XY}^{-}(q) \tag{17}$$

The larger the magnitude of $\Delta H_{XY}^{\pm}(q)$, the more pronounced the asymmetry of the cross-correlation, when the first market experienced different trends. The sign of $\Delta H_{XY}^{\pm}(q)$ indicates the dependence on the trending of the first market. If $\Delta H_{XY}^{\pm}(q) = 0$, the cross-correlation is symmetric for different trends of x. If $\Delta H_{XY}^{\pm}(q) > 0$, it means that the cross-correlation exponent is higher when the time series X has a positive trend than when it is negative. If $\Delta H_{XY}^{\pm}(q) < 0$, the cross-correlation exponent is lower when x has a positive trend than when it is negative.

3.1.2 Cross-correlation significance test

As a preliminary analysis, it is useful to examine the existence of cross-correlations qualitatively. To this end, Podobnik et *al*. (2009) developed the Q_{CC} statistic test. Suppose $(X_t)_{1\leq t\leq N}$ and $(Y_t)_{1\leq t\leq N}$ are two time series of length N. Podobnik et *al*. (2009) have defined the cross-correlation function C_i by : for $1 \leq i \leq N-1$

$$C_i = \frac{\sum_{k=i+1}^{N} X_k . Y_{k-1}}{\sqrt{\sum_{k=1}^{N} X_k^2 . \sum_{k=1}^{N} Y_k^2}} \tag{18}$$

The cross-correlation statistic Q_{CC} is defined by : for $1 \leq s \leq N-1$

$$Q_{CC}(s) = N^2 . \sum_{i=1}^{s} \frac{C_i^2}{N-s} \tag{19}$$

Podobnik et *al*. (2009) demonstrated that $Q_{CC}(s)$ is approximately $\chi^2(s)$ distributed with s degrees of freedom. The test can be used to test the null hypothesis that none of the first s cross-correlation coefficients is different from zero. The authors proposed to use the statistic by plotting the test statistic $Q_{CC}(s)$ versus the critical values $\chi^2(s)$ for a broad range of the degree of freedom s. If for a broad range of s the test statistic $Q_{CC}(s)$ deviates from the critical values at a 95% level of confidence, we can claim that cross-correlations are not only significant, but there are long-range cross-correlations. However, this test statistic, as a correlation coefficient, is a measure of linear cross-correlations, and as pointed by Podobnik *et al*. (2009), this cross-correlations test should be used to test the presence of cross-correlations only qualitatively.

3.1.3 DCCA cross-correlation coefficient

To assess quantitively the cross-correlations between two non-stationary series, Zebende (2011) proposed a DCCA cross-correlation coefficient. This method is based on the DCCA method of Podobnik and Stanley (2008) and the DFA method (Peng et al. 1994). In contrast to the original DCCA cross-correlation coefficient method, which uses non-overlapping segments, we will apply a version that uses overlapping segments, similar to the MF-DCCA method. Using the notations from section 3.2.1, the method is described below.

The covariance $f_{XY}^2(v,s)$ is defined in (8) and (9). The covariances $f_X^2(v,s)$, $f_Y^2(v,s)$ are defined by:

For $1 \leq v \leq Ns$:

$$f_X^2(v,s) = \frac{1}{s}\sum_{i=1}^{s}\left(X\left((v-1)s+i\right) - p_{X,v}^{m}(i)\right)^2 \tag{20}$$

$$f_Y^2(v,s) = \frac{1}{s}\sum_{i=1}^{s}\left(Y\big((v-1)s+i\big) - p_{Y,v}^m(i)\right)^2$$

and

For $Ns+1 \leq v \leq 2Ns$:

$$f_X^2(v,s) = \frac{1}{s}\sum_{i=1}^{s}\left(X\left(\big((N-v-Ns)s+i\big)\right) - p_{X,v}^m(i)\right)^2 \tag{21}$$
$$f_Y^2(v,s) = \frac{1}{s}\sum_{i=1}^{s}\left(Y\left(\big((N-v-Ns)s+i\big)\right) - p_{Y,v}^m(i)\right)^2$$

By averaging the covariances over all segments, we obtain the DFA-variance fluctuation functions $F_{DFA-X}^2(s)$ and $F_{DFA-Y}^2(s)$, and the DCCA-covariance fluctuation function $F_{DCCA}^2(s)$ defined by:

$$\begin{cases} F_{DFA-X}^2(s) = \dfrac{1}{2Ns}\displaystyle\sum_{v=1}^{2Ns} f_X^2(v,s) \\ F_{DFA-Y}^2(s) = \dfrac{1}{2Ns}\displaystyle\sum_{v=1}^{2Ns} f_Y^2(v,s) \\ F_{DCCA}^2(s) = \dfrac{1}{2Ns}\displaystyle\sum_{v=1}^{2Ns} f_{XY}^2(v,s) \end{cases} \tag{22}$$

The DCCA cross-correlation coefficient $\rho_{DCCA}(s)$ is defined by:

$$\rho_{DCCA}(s) = \frac{F_{DCCA}^2(s)}{\sqrt{F_{DFA-X}^2(s)} \times \sqrt{F_{DFA-Y}^2(s)}} \tag{23}$$

The cross-correlation coefficient $\rho_{DCCA}(s)$ is an effective measure with properties similar to those of the standard correlation coefficient. It is a dimensionless quantity that ranges from -1 to 1. When $\rho_{DCCA}(s) = 0$, there is no cross-correlation between the two series. A value of $-1 < \rho_{DCCA}(s) < 0$ indicates an anti-persistent cross-correlation, while $0 < \rho_{DCCA}(s) \leq 1$ suggests a persistent cross-correlation between the two series. If $\rho_{DCCA}(s) = -1$, the two series are perfectly anti-persistent cross-correlated. Conversely, if $\rho_{DCCA}(s) = 1$, the two series are perfectly persistent cross-correlated.

3.2 Data

The data consists of daily closing prices of Brent Oil Futures (BRENT), MASI index and MASI Oil & Gas sectorial index (MASI Oil). The data span from 04/01/2010 to 17/01/2025, comprising nearly 3915 observations. The selection of this period is based on both empirical relevance and methodological rigor. It represents the timeframe for which complete, consistent, and high-frequency (daily) data are available for all three variables under study—Brent Oil Futures, the MASI index, and the MASI Oil & Gas sector index—sourced from www.investing.com. Moreover, this period encompasses a series of major economic and geopolitical events that have significantly influenced oil and financial markets, including the post-2008 recovery, the 2014–2016 oil price collapse, the COVID-19 pandemic (2020), the Russia–Ukraine war (2022), and recent market developments extending into 2025.

All data were downloaded from the website www.investing.com. The prices were then converted into logarithmic returns $r_t = ln\left(\frac{P_t}{P_{t-1}}\right) = ln(P_t) - ln(P_{t-1})$, where P_t denotes the index daily price and ln corresponds to the natural logarithm.

4. Results and discussion

4.1 Result of cross-correlation significance test

In this section, we have checked qualitatively the presence of the cross-correlations between the Brent crude oil index and the two indices, the Masi index and the MASI oil sectorial index, using the Q_{CC} statistic. For the two pairs BRENT-MASI and BRENT-MASI OIL, we have plotted the decimal logarithm of the test statistic $Q_{CC}(s)$ versus the decimal logarithm of the critical values $\chi^2_{0,5}(s)$ at 5% significance level for a broad range of the degree of freedom s, $1 \leq s \leq 400$. The results are given in Figure 1.

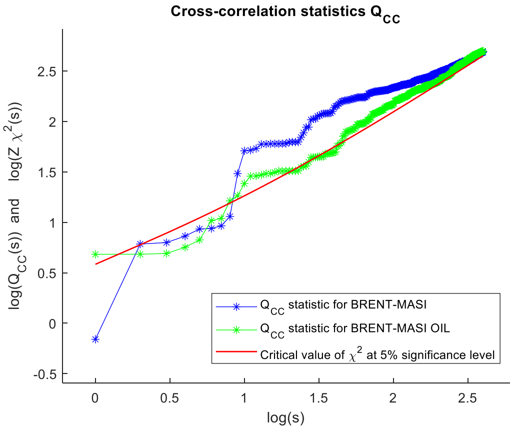

Figure 1: $Q_{CC}(s)$ and $Log(\chi^2_{0,5}(s))$ vs. $Log(s)$ for the pairs BRENT-MASI and BRENT-MASI OIL

We can observe that for sufficiently large values of s, the $Q_{CC}(s)$ statistics exceed the critical values $\chi^2_{0,5}(s)$, indicating that the cross-correlations are significant for the BRENT-MASI and BRENT-MASI OIL. However, this test statistic is considered as a linear and qualitative measure of cross-correlations. The results must be confirmed by the application of MF-DCCA method.

4.2 Result of the Asymmetric DCCA cross-correlation coefficient

We use the Asymmetric DCCA cross-correlation coefficient to quantify the asymmetrical cross-correlations between BRENT market and the MASI index as well as the MASI OIL index during various Brent market trends. Figure 2 depicts the overall, uptrend, and downtrend DCCA cross-correlation coefficients across various timescales, for BRENT- MASI and BRENT-MASI OIL.

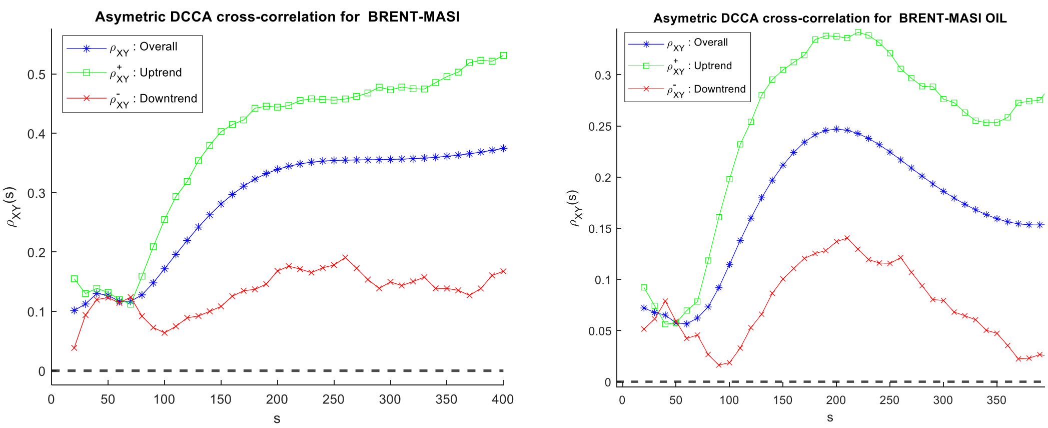

Figure 2: Overall, uptrend, and downtrend DCCA cross-correlation coefficients for BRENT-MASI and BRENT-MASI OIL

Figure 2 shows the clear distinction between the behavior of the uptrend $\rho^{+}_{DCCA}(s)$ and the downtrend $\rho^{-}_{DCCA}(s)$ of the BRENT- MASI and BRENT-MASI OIL pairs across all time scales, indicating asymmetric cross-correlations for the two pairs, depending on BRENT's upward and downward trends.

The cross-correlation between Brent crude oil (BRENT) and the Moroccan All Shares Index (MASI) highlights asymmetric market interconnectedness, where oil price movements influence the stock market depending on the trend direction. This is important for investors in oil-sensitive sectors like energy, transportation, and manufacturing. Stronger correlations during Brent's uptrend suggest that industries such as energy production or exports may benefit from rising oil prices, while downtrends might affect other sectors more, influencing sectoral investment strategies. Given Morocco's reliance on oil imports, investors should consider these asymmetries when making decisions about Moroccan stocks, particularly in oil-sensitive sectors.

For the BRENT-MASI OIL pair, the cross-correlation directly impacts oil and energy-related companies, with stronger and more direct correlations during Brent price movements. This makes hedging against oil price fluctuations more viable, although strategies should account for the asymmetric behavior depending on the direction of the trend. The sector tends to perform better during Brent's upward movements due to higher revenues from rising oil prices but may underperform during downtrends, offering cyclical investment opportunities within the oil and energy sector in Morocco.

4.3 Results of the MF-ADCCA

In this section, we apply MF-ADCCA to explore the multifractal asymmetric cross-correlation between BRENT market and the MASI index as well as the MASI OIL index during various Brent market trends, specifically uptrends and downtrends.

4.3.1 Multifractal and persistence behaviors

Figure 3 presents the generalized Hurst exponents, the Rényi exponents, and the singularity spectrum functions, as q varies from -45 to 45, respectively for BRENT- MASI and BRENT-MASI OIL.

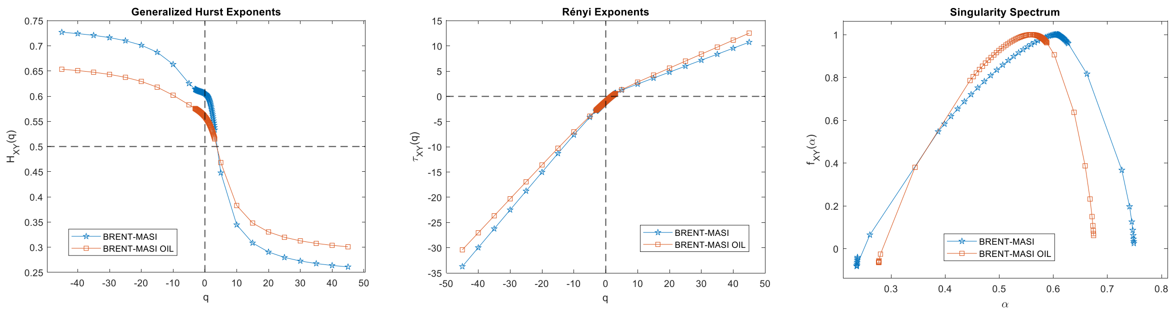

Figure 3: Generalized Hurst exponents, Rényi exponents and singularity spectrum for BRENT-MASI and BRENT-MASI OIL, for q varying from -45 to 45

Figure 3 shows that for the two pairs BRENT-MASI and BRENT-MASI OIL, $H_{XY}(q)$ decreases and $\tau_{XY}(q)$ increases non-linearly as q increases from -45 to 45, while the functions $f_{XY}(\alpha)$ exhibit inverted parabolic shapes. This indicates that the cross-correlations between BRENT market and the MASI index as well as the MASI OIL exhibit multifractal behaviors. This means that the correlation structures are complex and vary across different time scales. For practitioners, this highlights the need to account for time-varying correlations when managing risk or creating diversified portfolios that include both oil and stock market assets. The non-linear changes in $H_{XY}(q)$ and $\tau_{XY}(q)$ as q increases suggest that the degree of cross-correlation between BRENT and MASI markets depends on the magnitude of the fluctuations being observed. This means that during times of extreme market movements (both large gains or losses), the correlation dynamics might change, indicating the need for dynamic risk management strategies that can adapt to different market conditions.

Furthermore, for negative values of q, $H_{XY}(q) > 0.5$, with $H_{XY}(q)$ asymptotically tending to 0.73 for BRENT-MASI and 0.65 for BRENT-MASI OIL as q approaches -45. This indicates that during periods dominated by smaller fluctuations (associated with negative q), there is a persistent relationship between oil prices and the stock indices. This persistence suggests that these markets tend to move together over long periods when smaller market movements are considered. For portfolio diversification, this implies that relying on these markets as hedges for one another may not be effective, particularly during stable or low volatility periods.

The observed multifractality and persistence in cross-correlations also provide insights into market efficiency. The persistent behavior for negative q values suggests that there could be predictability in the relationship between oil and stock markets during periods of smaller price changes, which could be exploited by traders. However, the non-linearity of correlations across different magnitudes of fluctuations indicates that predicting these relationships may be challenging and require sophisticated, adaptive models.

Conversely, for positive values of q, $H_{XY}(q) < 0.5$, with $H_{XY}(q)$ asymptotically tending to 0.26 for BRENT-MASI and 0.30 for BRENT-MASI OIL as q approaches 45. This indicates antipersistent behavior in the cross-correlations of the two pairs BRENT-MASI and BRENT-MASI OIL. Antipersistence means that large movements in one market (BRENT or MASI, and BRENT or MASI-OIL) are likely to be followed by opposite movements in the other market. This suggests that during periods dominated by larger fluctuations (as captured by positive q), the markets tend to move in opposite directions. The specific asymptotic values of $H_{XY}(q)$ — 0.26 for BRENT-MASI and 0.30 for BRENT-MASI OIL as q approaches 45 — further indicate strong antipersistence for large positive q. These values being significantly lower than 0.5 suggest that the larger the fluctuations, the more strongly the markets move in opposite directions. This is important for understanding how the relationship between oil prices and the stock market behaves during times of market stress or large shocks. The antipersistent behavior for positive q suggests that large price changes in BRENT could be offset by opposite movements in the MASI and MASI-OIL indices. This could be useful for hedging purposes, as investors might exploit this negative correlation during periods of high volatility.

To measure the intensity of multifractality between BRENT market and the MASI index as well as the MASI OIL index, the metrics ΔH_{XY} and $\Delta\alpha_{XY}$ are used. The results are presented in Table 1.

Table 1: Intensity of multifractality for the pairs BRENT-MASI and BRENT-MASI OIL

Range	Pair	ΔH_{XY}	$\Delta\alpha_{XY}$
	BRENT-MASI	0.466	0.512
	BRENT-MASI OIL	0.353	0.397

Table 1 shows that $\Delta H_{XY} \neq 0$ and $\Delta\alpha_{XY} \neq 0$, confirming that all the cross-correlations of BRENT versus MASI and the MASI-OIL indices exhibit multifractal characteristics. The highest values of ΔH_{XY} and $\Delta\alpha_{XY}$ for the BRENT-MASI suggests strong and persistent multifractal behavior between BRENT and MASI, indicating that their cross-correlation is notably complex and exhibits a high level of sensitivity across different timescales. The cross-correlation between BRENT and MASI-OIL also demonstrates strong multifractality, although slightly weaker than that of BRENT-MASI.

4.3.2 Analysis of asymmetry using fluctuation functions in the monofractal case

Figure 4 presents the log-log plots of the overall $F_2(s)$, uptrend $F_2^+(s)$ and downtrend $F_2^-(s)$ fluctuation functions versus s, for BRENT against the MASI index and MASI OIL index, under different BRENT market conditions.

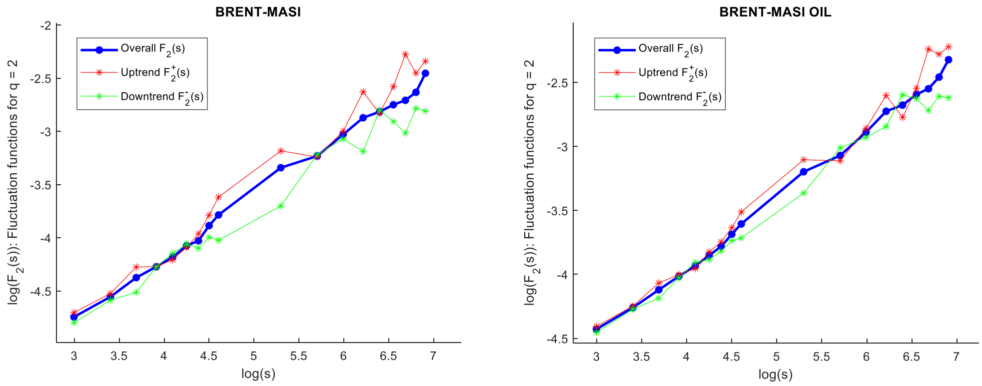

Figure 4: Log-log plots of the overall, uptrend and downtrend fluctuation functions versus s for BRENT against the MASI and MASI OIL indices, under different BRENT market trends (case $q = 2$)

The graphs above show a distinction in the behaviors of $F_2^+(s)$ and $F_2^-(s)$ across all timescales four BRENT against the MASI and MASI OIL indices. This indicates the presence of asymmetry in the cross-correlations for the two pairs BRENT-MASI and BRENT-MASI OIL.

To quantify more precisely this asymmetric behavior, we use the metric $\Delta log\left(F_2^{\pm}(s)\right) = log\left(F_2^+(s)\right) - log\left(F_2^-(s)\right)$. Figure 5 shows the results obtained.

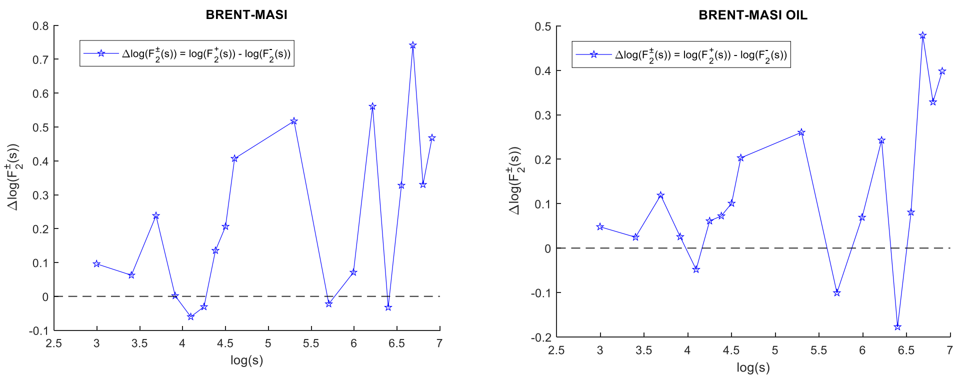

Figure 5: Difference in fluctuations between uptrend $F_2^+(s)$ and downtrend $F_2^-(s)$ for BRENT-MASI and BRENT-MASI OIL

Recall that if $\Delta log\left(F_2^{\pm}(s)\right) = 0$, there is a symmetry, and the larger the absolute value of this metric, the stronger the asymmetry. The graphs above show that the fluctuation excesses between uptrend $F_2^+(s)$ and downtrend $F_2^-(s)$ for BRENT-MASI and BRENT-MASI OIL reveal large amplitude variations around zero, reinforcing the existence of asymmetry in all the cross-correlations over various time scales. This indicates that the relationships between BRENT and MASI index as well as MASI OIL are not the same in bullish and bearish market conditions of BRENT.

4.3.3 Analysis of asymmetry intensity using generalized Hurst exponents

Figure 6 displays the overall $H_{XY}(q)$, uptrend $H_{XY}^+(q)$, and downtrend $H_{XY}^-(q)$ generalized cross-correlation exponents for BRENT-MASI and BRENT-MASI OIL, versus q varying from −45 to 45.

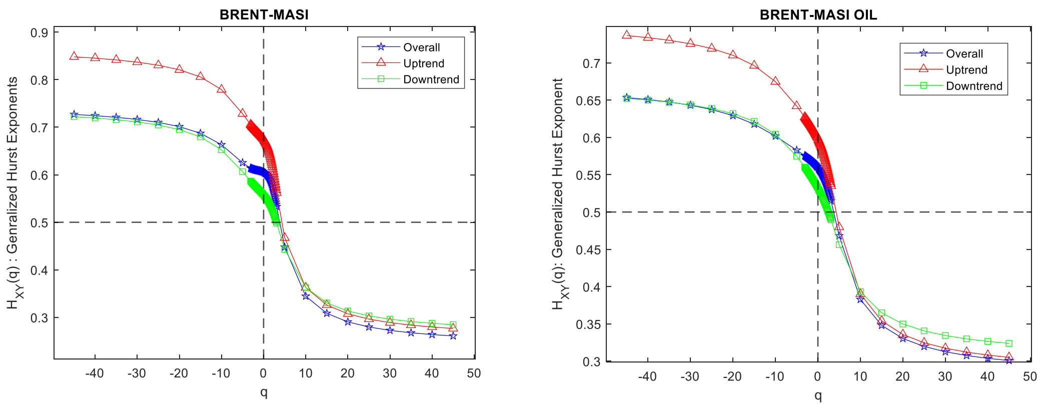

Figure 6: Plots of the overall, uptrend and downtrend generalized Hurst exponents for BRENT-MASI and BRENT-MASI OIL, for q varying from -45 to 45

Figure 6 shows that BRENT-MASI and BRENT-MASI OI, $H_{XY}^{+}(q)$ and $H_{XY}^{-}(q)$ decrease in a similar non-linear manner as q increases from -45 to 45. This indicates that all the cross-correlations exhibit multifractal characteristics during both bullish and bearish market conditions of BRENT.

Furthermore, for negative values of q, the uptrend $H_{XY}^{+}(q)$ for the BRENT-MASI and BRENT-MASI OIL pairs deviates significantly above the overall $H_{XY}(q)$ as q approaches -45, while the downtrend $H_{XY}^{-}(q)$ closely aligns with $H_{XY}(q)$. This suggests that during periods of negative fluctuations, the cross-correlations in the BRENT-MASI and BRENT-MASI OIL pairs are stronger or more persistent when the BRENT market is experiencing bullish conditions compared to when it is in a bearish phase. Thus, for negative fluctuations ($q < 0$) the cross-correlations between BRENT and MASI as well as MASI OIL differ under the bullish and bearish market conditions of BRENT. Consequently, the cross-correlations for the BRENT-MASI and BRENT-MASI OIL pairs exhibit asymmetry depending on whether the BRENT market is in a bullish or bearish phase.

For positive values of q, $H_{XY}^{+}(q)$ and $H_{XY}^{-}(q)$ are nearly identical and slightly above $H_{XY}(q)$ for BRENT-MASI. In contrast, for BRENT-MASI OIL, $H_{XY}^{+}(q)$ closely follows $H_{XY}(q)$, while $H_{XY}^{-}(q)$ is slightly above $H_{XY}(q)$. This suggests that during periods of positive fluctuations, the cross-correlation in the BRENT-MASI pair behaves in a similar manner in bullish and bearish conditions, demonstrating symmetry. For investors, this stability can be useful for formulating strategies that are less sensitive to market trends. When BRENT is experiencing growth, the correlation with MASI is predictable, offering a more reliable foundation for investment decisions or hedging strategies. While, For the BRENT-MASI OIL pair, the cross-correlation exhibits asymmetry depending on whether the BRENT market is in a bullish or bearish phase.

For positive values of q, $H_{XY}^{+}(q)$ and $H_{XY}^{-}(q)$ for the BRENT-MASI pair are nearly identical and slightly higher than $H_{XY}(q)$. In contrast, for BRENT-MASI OIL, $H_{XY}^{+}(q)$ closely follows $H_{XY}(q)$, while $H_{XY}^{-}(q)$ is slightly above it. This indicates that during periods of positive fluctuations, the cross-correlation between BRENT and MASI behaves similarly under both bullish and bearish conditions, showing a symmetric pattern. This stability can help investors design strategies that are less affected by market trends, as the correlation between BRENT and MASI remains consistent during market growth. On the other hand, the cross-correlation for the BRENT-MASI OIL pair displays asymmetry, varying depending on whether the BRENT market is in a bullish or bearish phase.

Figure 7 displays the values of the asymmetric degree of cross-correlations measured by the excess differences $\Delta H_{XY}^{\pm}(q)$ for the BRENT-MASI and BRENT-MASI OIL pairs.

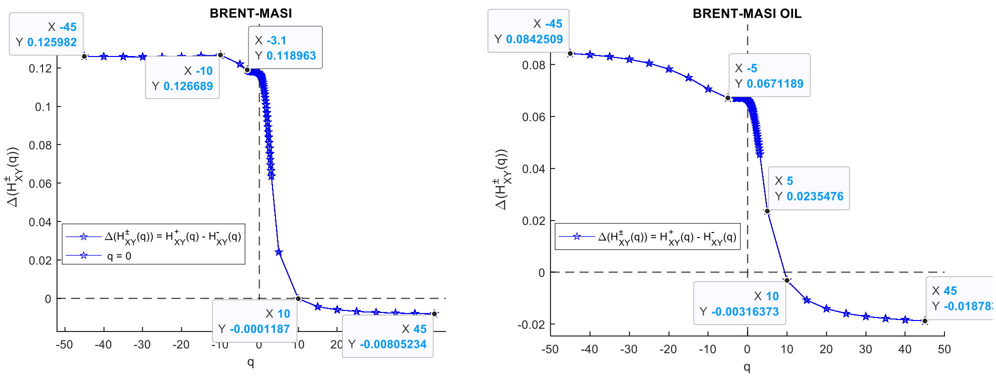

Figure 7: Asymmetric degree of cross-correlations BRENT-MASI and BRENT-MASI OIL

▪ *Interpretation of the cross-correlations of the* BRENT-MASI pair*:*

For $q < 0$ (negative fluctuations), $\Delta H_{XY}^{\pm}(q)$ remains positive, indicating that $H_{XY}^{+}(q)$ is larger than $H_{XY}^{-}(q)$. This suggests that the correlation between BRENT and MASI exhibits long-range persistence during BRENT's bullish market conditions. In this range, the cross-correlation between BRENT and MASI is asymmetric varying depending on the specific trend of BRENT. Notably, for q between -45 and -10, $\Delta H_{XY}^{\pm}(q)$ remains nearly constant at 0.126. As q increases from -10 to 45, $\Delta H_{XY}^{\pm}(q)$ gradually decreases to 0, indicating a shift towards symmetry. Specifically, for positive q values increasing from 10 to 45, $\Delta H_{XY}^{\pm}(q) \approx 0$, confirming symmetry.

The implications of this analysis are significant for investors and market analysts. The positive values of $\Delta H_{XY}^{\pm}(q)$ for $q < 0$ suggest that, during periods of negative fluctuations, the correlation between BRENT and MASI is stronger during BRENT's bullish phases compared to its bearish ones. This long-range persistence in bullish conditions can provide opportunities for investors who seek to leverage long-term trends when BRENT is performing well, as the correlation with MASI remains consistently high.

The asymmetric cross-correlation for negative q values implies that the relationship between BRENT and MASI varies based on the specific trend of BRENT, making it crucial for market participants to account for the current market phase when making portfolio decisions or risk assessments. This asymmetry may also indicate that the two markets react differently to positive and negative market events, affecting strategies like hedging or diversification.

As q increases (positive fluctuations), $\Delta H_{XY}^{\pm}(q)$ approaches zero, signaling a shift toward symmetry. For investors, this suggests that during periods of positive fluctuations, the correlation between BRENT and MASI becomes more stable and predictable, regardless of BRENT's market trend. This symmetry could simplify investment strategies during bullish conditions, as the cross-correlation remains consistent, reducing the risk of unexpected market behavior.

✓ Interpretation of the cross-correlations of the BRENT-MASI OIL pair:

For $q < 0$ (negative fluctuations), $\Delta H_{XY}^{\pm}(q)$ remains positive, indicating that $H_{XY}^{+}(q)$ is larger than $H_{XY}^{-}(q)$. This suggests that the correlation between BRENT and MASI OIL exhibits long-range persistence during BRENT's bullish market conditions. In this range, the cross-correlation between BRENT and MASI OIL is asymmetric varying depending on the specific trend of BRENT. Notably, for q between -45 and -5, $\Delta H_{XY}^{\pm}(q)$ decreases from 0.084 to 0.067. As q increases from -5 to 10, $\Delta H_{XY}^{\pm}(q)$ gradually decreases to -0.003 close to 0, indicating a shift towards symmetry. When q increases from 10 to 45, $\Delta H_{XY}^{\pm}(q)$ tends asymptotically to -0.019.

The practical implications of this analysis for the BRENT-MASI OIL pair are quite insightful for investors and market participants.

For $q < 0$ (negative fluctuations), $\Delta H^{\pm}_{XY}(q)$ remains positive, indicating that the cross-correlation between BRENT and MASI OIL is stronger during bullish phases of BRENT compared to bearish ones. This suggests that during BRENT's uptrend, investors can expect a more persistent relationship between the two markets. This information is useful for portfolio managers and traders who might want to exploit long-range correlation during bullish conditions, as it highlights a strong co-movement between oil prices and related markets.

As q increases from -45 to -5, the cross-correlation asymmetry gradually reduces, implying that the degree of correlation depends on the extent of fluctuations and the specific trend of BRENT. The decrease from 0.084 to 0.067 signals that, while there is still a positive correlation, the strength of the relationship is waning. Investors could interpret this as an opportunity to monitor the market closely, as weaker correlations during high volatility periods might signal upcoming changes or reversals in market behavior.

As q increases beyond 10, $\Delta H^{\pm}_{XY}(q)$ tends asymptotically towards -0.019. This slight negative value suggests a small asymmetry in favor of bearish trends for strong positive fluctuations in BRENT. Investors should be aware that, during periods of significant market rises, MASI OIL may slightly underperform or respond differently compared to its behavior during moderate fluctuations. This subtle shift could be important for risk management and hedging strategies, especially in periods of strong market trends.

4.4 Comparison with previous studies

This study on the relationship between Brent crude oil market trends and the Moroccan stock market using Multifractal Detrended Asymmetric Cross-Correlation Analysis (MF-ADCCA) presents significant findings that align with and extend the results of existing literature on the oil-stock market nexus, particularly in oil-importing economies.

Our findings revealed substantial long-range dependencies and significant asymmetric cross-correlations between Brent crude oil prices and the Moroccan stock market indices (MASI and MASI OIL). These results are consistent with studies using traditional econometric models, such as VAR and VECM, which have demonstrated that oil price shocks can influence stock market returns with varying degrees of persistence. For example, Cuñado and Pérez de Gracia (2014) and Filis et al. (2011) observed significant correlations between oil prices and stock returns in oil-importing European economies, where supply-side shocks were found to have a more pronounced effect. In our study, the persistence and asymmetric nature of these correlations echo these findings, reinforcing the view that oil price fluctuations exert considerable influence on stock markets, especially in oil-dependent economies.

The MF-ADCCA approach used in this study allowed for a deeper exploration of the multifractal nature of the cross-correlations between Brent and the Moroccan indices. Our results confirmed the presence of multifractality across all orders of q, highlighting the complex, scale-dependent nature of these relationships. This aligns with findings from Burugupalli (2015) and Adekoya et al. (2023), who employed MF-DCCA to uncover multifractal correlations between oil prices and various financial assets, such as gold and European stock markets. Yao et al. (2020) also identified persistent multifractal correlations between oil and stock markets, further supporting the notion that oil price shocks trigger nonlinear, persistent, and multifractal behavior in financial markets.

The asymmetric nature of cross-correlations, particularly under different market conditions (bullish and bearish), is another key result of our study. The uptrend and downtrend generalized Hurst exponents displayed distinct patterns, reflecting the varying dynamics of the Brent-MASI and Brent-MASI OIL relationships. This is in line with research by Gajardo and Kristjanpoller (2017), who applied MF-ADCCA to Latin American and U.S. stock markets and found asymmetric cross-correlations with crude oil prices. Their study highlighted that the intensity and direction of cross-correlations varied depending on the market trends, a result that mirrors the asymmetry observed in our analysis.

In conclusion, our study extends the existing body of research by offering a comprehensive analysis of the multifractal and asymmetric cross-correlations between Brent crude oil prices and the Moroccan stock market. By using MF-ADCCA, we provide a nuanced understanding of the complex dynamics between oil prices and stock markets, complementing and building upon traditional econometric studies and multifractal analyses. The results emphasize the need for more sophisticated approaches to forecasting and managing risks in emerging markets, where oil price fluctuations play a pivotal role in shaping financial market outcomes.

5. Conclusion

This study explores the impact of Brent crude oil market trends on the Moroccan stock market by employing Multifractal Detrended Asymmetric Cross-Correlation Analysis (MF-ADCCA). As an initial step, we assessed the existence of cross-correlations by calculating the Q_CC Cross-Correlation statistics for the Brent-MASI and Brent-MASI OIL pairs. The results revealed significant long-range dependencies, indicating a high degree of interconnectedness between Brent and both indices.

Furthermore, the Asymmetric DCCA cross-correlation coefficients indicated asymmetric and persistent cross-correlations across various time scales, reinforcing the presence of long-range dependencies between Brent and the MASI/MASI OIL indices under varying market conditions.

In the application of MF-ADCCA, we analyzed the generalized Hurst exponents, Rényi exponents, and singularity spectrum functions. The findings showed that the cross-correlations between Brent and the two indices exhibited multifractal behavior across all orders of q, signifying the complex and multifaceted nature of their interactions.

In the monofractal case, the uptrend and downtrend fluctuation functions demonstrated distinct behavior across all time scales, highlighting the presence of asymmetry in the cross-correlations of Brent-MASI and Brent-MASI OIL. The analysis of the uptrend and downtrend generalized Hurst exponents further illustrated nonlinear decreases throughout the fluctuation range, signifying multifractal characteristics during both bullish and bearish market phases.

The graphical analysis confirmed that the uptrend and downtrend generalized exponents exhibited distinct patterns, surrounding the overall generalized Hurst exponent across fluctuation orders, indicating that correlations between Brent and the indices vary depending on the direction of the Brent market trend. This emphasizes the asymmetric nature of the cross-correlations in the Brent-MASI and Brent-MASI OIL pairs, contingent on market conditions.

The practical implications of these findings are significant for investors, fund managers, and policymakers, offering valuable insights into the impact of Brent crude oil market trends on the Moroccan stock market, specifically the MASI and MASI OIL indices.

For investors and fund managers, the understanding of multifractal and asymmetric cross-correlations between Brent and the Moroccan indices can greatly improve investment strategies. By adjusting portfolios based on oil price fluctuations, investors can better navigate both bullish and bearish market conditions. Furthermore, dynamic hedging strategies using Brent futures or related derivatives can be implemented to mitigate risks arising from oil price volatility, thus enhancing risk-adjusted returns.

For policymakers and regulators, these findings highlight the vulnerability of the Moroccan stock market to oil price volatility. Policymakers can use this knowledge to strengthen market resilience by adopting counter-cyclical policies and proactive measures such as enhancing liquidity and adjusting monetary policies to safeguard financial stability during oil price shocks.

Additionally, understanding the asymmetric nature of the cross-correlations can help regulators forecast how different market conditions might unfold, allowing for better policy responses during periods of oil price volatility.

For Morocco, as an oil-importing country, these findings carry additional importance due to the country's sensitivity to global oil price fluctuations. Given the dependence on imported crude oil, understanding the multifractal and asymmetric relationships between Brent crude oil prices and the Moroccan stock market can help Morocco better anticipate and manage the economic consequences of oil price volatility.

In conclusion, this study underscores the importance of closely monitoring global oil price trends and understanding their multifaceted effects on Morocco's economy and financial markets. The insights gained from this analysis equip policymakers with the tools to implement more effective economic policies, strengthen market resilience, and reduce vulnerability to oil price volatility. These findings contribute to a deeper understanding of the multifractal and asymmetric nature of cross-correlations between Brent and the MASI/MASI OIL indices, offering valuable insights for investors, policymakers, and researchers alike.

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